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CREEP OF 2618 ALUMINUM UNDER STEP STRESS CHANGES PREDICTED BY A--ETC(U)
APR 79 J S LAI, W N FINDLEY DAAG29-78-G-0185

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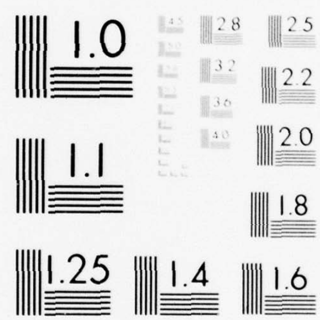

MICROCOPY RESOLUTION TEST CHART
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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

12

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: 19 15508.1-E 18 ARO	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): 6 CREEP OF 2618 ALUMINUM UNDER STEP STRESS CHANGES PREDICTED BY A VISCOUS-VISCOELASTIC MODEL.	5. TYPE OF REPORT & PERIOD COVERED: 9 Technical rept.
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): 10 James S. Lai, William N. Findley	8. CONTRACT OR GRANT NUMBER(s): 15 DAAG29-78-G-0185 new
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Brown University, Providence, Rhode Island	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 12 29p.
11. CONTROLLING OFFICE NAME AND ADDRESS: U. S. Army Research Office, P. O. Box 12211, Research Triangle Park, NC 27709	12. REPORT DATE: 11 Apr 79
	13. NUMBER OF PAGES: 25
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

LEVEL IV

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

14 EMRL-71, TR-1

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JUN 26 1979
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17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

viscous-viscoelastic modeling
torsion stresses
strain hardening theory
constitutive equations
aluminum creep behavior
tension stresses

405 355

JOB

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Nonlinear constitutive equations are developed and used to predict from constant stress data the creep behavior of 2618 Aluminum at 200°C (392°F) for tension or torsion stresses under varying stress history including step-up, step-down, and reloading stress changes. The strain in the constitutive equation employed includes the following components: linear elastic, time-independent plastic, nonlinear time-dependent recoverable (viscoelastic), nonlinear time-dependent nonrecoverable (viscous) positive, and nonlinear time-dependent nonrecoverable (viscous) negative. The modified superposition principle, derived from the multiple integral representation, and strain hardening theory were used

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Unclassified

20. ABSTRACT CONTINUED

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15508.1-E

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CREEP OF 2618 ALUMINUM UNDER STEP STRESS CHANGES PREDICTED BY A VISCOUS-VISCOELASTIC MODEL

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Army Research Office
Technical Report No. 1
Grant DAAG29-78-G-0185

ARO DAAG29-78-G-0185/1
EMRL-71

April 1979

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ABSTRACT

Nonlinear constitutive equations are developed and used to predict from constant stress data the creep behavior of 2618 Aluminum at 200°C (392°F) for tension or torsion stresses under varying stress history including step-up, step-down, and reloading stress changes.

The strain in the constitutive equation employed includes the following components: linear elastic, time-independent plastic, nonlinear time-dependent recoverable (viscoelastic), nonlinear time-dependent nonrecoverable (viscous) positive, and nonlinear time-dependent nonrecoverable (viscous) negative. The modified superposition principle, derived from the multiple integral representation, and strain hardening theory were used to represent the recoverable and nonrecoverable components, respectively, of the time-dependent strain in the constitutive equations. Excellent to fair agreement was obtained between the experimentally measured data and the predictions based on data from constant-stress tests using the constitutive equations as modified.

INTRODUCTION

The creep behavior of metals under changing stress--especially changes in state of combined stress and stress reversal--has received little experimental observation. Mathematical expressions employed, such as strain hardening or viscoelastic models, usually are unable to describe the detail of creep behavior under changes such as the above. References to prior work in this area are given in [1].

In a previous paper [1] the authors described a viscous-viscoelastic model in which the strain was resolved into five components: elastic ε^e, time-independent plastic ε^p, positive nonrecoverable (viscous) ε^v_{pos}, negative nonrecoverable (viscous) ε^v_{neg}, and recoverable (viscoelastic) ε^{ve} components. From creep and recovery experiments under combined tension and torsion, the time and stress dependence of these components were evaluated for constant stresses. Constitutive relations for changes in stress state also were discussed in [1].

In the present paper, constitutive equations for changes in state of combined tension and torsion are developed and used to predict, from the relations determined from constant stress tests in [1], the creep behavior under abrupt step-up and step-down changes in tension or torsion. The results are compared with experiments reported in [2] and with new experiments described in the following pages. Future work will consider abrupt changes in the state of combined tension and torsion, stress reversal, relaxation and simultaneous creep and relaxation.

MATERIAL AND SPECIMENS

An aluminum forging alloy 2618-T61 was employed in these experiments. Specimens were taken from the same lot of 2-1/2 in. diameter forged rod as used in [1] and the same lot as specimens D through H in [2]. Specimens were thin-walled tubes having outside diameter, wall thickness and gage length of 1.00, 0.060 and 4.00 inches respectively. A more complete description of material and specimens is given in [1].

EXPERIMENTAL APPARATUS AND PROCEDURE

The combined tension and torsion creep machine used for these experiments was described in [3] and briefly in [1]. The temperature control and measurement employed was described in [1,2]. Stress was produced by applying dead weights at the end of levers. These weights were applied by hand at the start of a test by lowering them quickly but without shock. The time of the start of the test was taken to be the instant at which the load was fully applied. In the present experiments changes in loading were made at intervals during the creep tests. The load changes were accomplished by hand in the same manner. Strain was recorded at the following intervals following a load change: every 0.01 h to 0.05 h; every 0.02 h to 0.1 h; every 0.05 h to 0.5 h; every 0.1 h to 1.0 h; and every 0.2 h to 2.0 h. All experiments were performed at 200°C (392°F).

CONSTITUTIVE EQUATIONS FOR CONSTANT STRESS

In this paper as in the previous one [1] the strain was resolved into five components: ε^e , ε^p , ε^v_{pos} , ε^v_{neg} , and ε^{ve} as defined in the introduction. The elastic strain ε^e was determined from the elastic constants at the test temperature. In [1] the elastic constants at 200°C (392°F) were determined indirectly from creep test data with the following results

$$E = 6.5 \times 10^4 \text{ MPa } (9.43 \times 10^6 \text{ psi}) \quad ,$$

$$G = 2.46 \times 10^4 \text{ MPa } (3.57 \times 10^6 \text{ psi}) \quad ,$$

$$\nu = 0.321 \quad ,$$

where E , G , and ν are the elastic modulus, shear modulus and Poisson's ratio respectively.

As noted in [1] plastic strains ϵ^p were essentially zero in the creep tests performed and creep at constant stress was well represented by a power function of time

$$\epsilon_{ij} = \epsilon_{ij}^{o} + \epsilon_{ij}^{+} t^n \quad , \tag{1}$$

where the time-independent strain ϵ_{ij}^{o} and the coefficient of the time-dependent strain terms ϵ_{ij}^{+} were functions of stress and n was a constant. It was also shown in [1] that the nonrecoverable $\epsilon^v(t)$ and recoverable $\epsilon^{ve}(t)$ components of time-dependent strain could each be represented by a power function of time with the same exponent n . Also it was shown that the ratio R of the coefficient of the recoverable time-dependent and nonrecoverable time-dependent strains could be taken as a constant. Thus, under a constant stress

$$\epsilon_{ij}^{v} = [1/(1+R)]\, \epsilon_{ij}^{+} t^n \quad , \tag{2}$$

$$\epsilon_{ij}^{ve} = [R/(1+R)]\, \epsilon_{ij}^{+} t^n \quad . \tag{3}$$

In the previous work [1], the authors found the time-dependent strain of the material under single step loading and recovery to be well described by the following two equations for time-dependent pure axial strain ϵ_{11}^{+} and pure shear strain ϵ_{12}^{+}.

$$\epsilon_{11}^{+}(\sigma) = F(\sigma) = F_1^{+}(\sigma-\sigma^*) + F_2^{+}(\sigma-\sigma^*)^2 + F_3^{+}(\sigma-\sigma^*)^3 \quad , \tag{4}$$

$$\varepsilon_{12}^{+}(\tau) = G(\tau) = G_1^{+}(\tau-\tau^*) + G_2^{+}(\tau-\tau^*)^3 \quad . \tag{5}$$

The nonlinear relationship of σ, and τ in ε_{11}^{+} and ε_{12}^{+} was derived from a third order multiple integral representation [4,5]. In (4) and (5) σ^*, τ^* are the creep limits in pure tension and pure torsion respectively, where $\sigma-\sigma^*$ or $\tau-\tau^*$ are zero for $-\sigma^* \leq \sigma \leq \sigma^*$ or $-\tau^* \leq \tau \leq \tau^*$ respectively. The creep limit defines a stress below which creep appears to be zero or very small.†

Separating nonrecoverable ε^{v} and recoverable ε^{ve} strain components according to (2), (3) and using (4), (5) the time-dependent parts ε^{v} and ε^{ve} for creep under constant tension σ and torsion τ can be represented by the following equations:

$$\varepsilon_{11}^{ve}(t) = \left(\frac{R}{1+R}\right) F(\sigma-\sigma^*)\ t^n \quad , \tag{6}$$

$$\varepsilon_{12}^{ve}(t) = \left(\frac{R}{1+R}\right) G(\tau-\tau^*)\ t^n \quad , \tag{7}$$

$$\varepsilon_{11}^{v}(t) = \left(\frac{1}{1+R}\right) F(\sigma-\sigma^*)\ t^n \quad , \tag{8}$$

$$\varepsilon_{12}^{v}(t) = \left(\frac{1}{1+R}\right) G(\tau-\tau^*)\ t^n \quad , \tag{9}$$

where F_i^{+}, G_i^{+}, σ^*, τ^*, R and n are the values determined from constant tension and torsion creep tests as reported earlier [1] and shown in Table I.

The rationale for separating the time-dependent strains into nonrecoverable strain ε^{v} and recoverable strain ε^{ve} was based on the assumption that recovery resulted from recoverable strain accumulated during creep. Thus ε^{ve} was determined from recovery data for the material in a set of constant stress

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†Information obtained after completion of this work indicated that there was creep below the creep limit, but at a much lower rate than above the creep limit.

creep and recovery tests as reported in [1]. ε^{v} was determined from creep tests by subtracting strains due to ε^{ve} as described in [1]. Under time-dependent stress inputs, including step changes, other considerations are required in addition to (6)-(9) for predicting ε^{v} and ε^{ve}. These considerations will be presented in the next section.

CONSTITUTIVE EQUATIONS FOR VARIABLE STRESS

Creep behavior is dependent on the past history of stress (or strain). History dependence can be incorporated in the multiple integral representation [4,5] for a recoverable-type material. Unfortunately, the experimental difficulty of determining F_i, G_i to completely characterize a given material is almost insurmountable [5]. Furthermore, as pointed out by Wang and Onat [6,7], higher order terms beyond the third order of the multiple integral representation appeared to be required to describe creep of metals under multiple step loadings with sufficient accuracy. In the following, constitutive equations are developed to describe ε^{v} and ε^{ve} under time-dependent stress history.

Constitutive Equation for ε^{ve}:

In [5], it was shown that the multiple integral representation and various simplified forms can be used to describe creep behavior of recoverable type material under variable stress. Among the various simplified forms, the modified superposition principle (MSP) [5] has been shown to yield satisfactory results. Thus, the modified superposition principle will be used here to describe the time-dependent recoverable strain ε^{ve}.

The modified superposition principle has the effect of reducing multiple integrals to single integrals. The modified superposition principle considers that following the first change in stress at time t_1 from σ_1 to σ_2 the creep strain is the sum of: the strain which would have resulted had the original stress σ_1 continued unchanged; plus the strain (negative) which would have resulted from an equal but opposite stress $(-\sigma_1)$ applied at t_1 to an untested specimen; plus the strain which would have resulted from applying the new stress σ_2 at t_1 to an untested specimen. Thus, if the strain at constant stress is given by

$$\varepsilon = f(\sigma,t) \tag{10}$$

the strain from N step changes in stress from σ_{i-1} to σ_i at time t_i is given by

$$\varepsilon(t) = \sum_{i=0}^{N} [f(\sigma_i,t-t_i) - f(\sigma_{i-1},t-t_{i-1})] \quad . \tag{11}$$

The modified superposition principle for a continuously varying stress may be expressed as follows by considering the limiting case as the steps in (11) tend to an infinite number of infinitesimal steps of stress,

$$\varepsilon(t) = \int_0^t \frac{\partial f[\sigma(\xi),t-\xi]}{\partial\sigma(\xi)} \dot{\sigma}(\xi)\, d\xi \quad . \tag{12}$$

Applying (11) to the following series of three steps in tension σ (or torsion τ) stress:

$\sigma_1(\tau_1)$ for $0 < t < t_1$, $\sigma_2(\tau_2)$ for $t_1 < t < t_2$ and $\sigma_3(\tau_3)$ for $t_2 < t$

yields the following by inserting (6) and (7) in (11). The time-dependent recoverable strain ε^{ve} following the third step is given by

$$\begin{aligned}\varepsilon^{ve}_{11}(t) = (\tfrac{R}{1+R})\{&F(\sigma_1)[t^n-(t-t_1)^n] \\ &+ F(\sigma_2)[(t-t_1)^n-(t-t_2)^n] \\ &+ F(\sigma_3)(t-t_2)^n\} \quad , \qquad t_2 < t\end{aligned} \tag{13}$$

$$\varepsilon^{ve}_{12}(t) = (\frac{R}{1+R})\{G(\tau_1)[t^n-(t-t_1)^n]$$

$$+ G(\tau_2)[(t-t_1)^n-(t-t_2)^n] \tag{14}$$

$$+ G(\tau_3)(t-t_2)^n\} \quad , \qquad t_2 < t \quad ,$$

where the stress functions $F(\sigma_i)$ and $G(\tau_i)$ represent $F(\sigma_i-\sigma^*)$ and $G(\tau_i-\tau^*)$ and are given in (4) and (5).

For a series of m steps in stress the shearing strain ε^{ve}_{12}, for example, following the m-th step, has the form:

$$\varepsilon^{ve}_{12}(t) = (\frac{R}{1+R})\{G(\tau_1)[t^n-(t-t_1)^n] + \ldots$$

$$+ G(\tau_{m-1})[(t-t_{m-2})^n-(t-t_{m-1})^n] \tag{15}$$

$$+ G(\tau_m)(t-t_{m-1})^n\} \quad , \qquad t_{m-1} < t \quad .$$

Now, if σ_1 , σ_2 (or τ_1 , τ_2) are greater than σ^* (or τ^*) respectively and if σ_3 (or τ_3) in the third step is less than the stresses σ_2 (or τ_2) respectively in the second step, then according to (13) [or (14)] both ε^{ve}_{11} and ε^{ve}_{12} will show partial recovery if $\sigma_3 > \sigma^*$ (or $\tau_3 > \tau^*$). **Also, whenever $\sigma_3 \leq \sigma^*$ (or $\tau_3 \leq \tau^*$) (including $\sigma_3 = \tau_3 = 0$), then the time-dependent strains will exhibit the same recovery as from complete unloading. The validity of this prediction will be explored later in this paper.**

Constitutive Equation for ε^V:

Strain hardening is taken to be applicable to the nonrecoverable strain. The relations employed were derived as follows. Consider the axial strains as an example. The derivative of (8) yields the axial strain rate $\dot{\varepsilon}^V_{11}(t)$

$$\dot{\varepsilon}^V_{11}(t) = \frac{n}{1+R} F(\sigma)\, t^{n-1}. \tag{16}$$

Eliminating t between (8) and (16) yields

$$\frac{\varepsilon_{11}^{v}(1+R)}{F(\sigma)} = \left[\frac{\dot{\varepsilon}_{11}^{v}(1+R)}{nF(\sigma)}\right]^{n/(n-1)} ,$$

from which

$$\frac{\dot{\varepsilon}_{11}^{v}}{(\varepsilon_{11}^{v})^{1-(1/n)}} = n\left[\frac{F(\sigma)}{1+R}\right]^{1/n} . \tag{17}$$

Multiplying both sides of (17) by dt and integrating yields

$$\varepsilon_{11}^{v}(t) = \frac{1}{1+R}\left[\int_{0}^{t}\{F[\sigma(\xi)]\}^{1/n}\, d\xi\right]^{n} . \tag{18}$$

In (18) it has been assumed, in accordance with the usual strain hardening concept, that the same function $F(\sigma)$ applies for variable stress $F[\sigma(\xi)]$ as for constant stress $F[\sigma]$.

For step changes in stress, such as the series of three steps given above, the axial strain in the third step may be found from (18) by employing the Dirac delta function as follows:

$$\begin{aligned}\varepsilon_{11}^{v}(t) = \frac{1}{1+R}\{[F(\sigma_1)]^{1/n}(t_1) + [F(\sigma_2)]^{1/n}(t_2-t_1)\\ + [F(\sigma_3)]^{1/n}(t-t_2)\}^{n} , \qquad t_2 < t .\end{aligned} \tag{19}$$

Similarly the shearing strain $\varepsilon_{12}^{v}(t)$ may be found as follows

$$\begin{aligned}\varepsilon_{12}^{v}(t) = \frac{1}{1+R}\{[G(\tau_1)]^{1/n}(t_1) + [G(\tau_2)]^{1/n}(t_2-t_1)\\ + [G(\tau_3)]^{1/n}(t-t_2)\}^{n} , \qquad t_2 < t .\end{aligned} \tag{20}$$

For a series of m steps in stress, the shearing strain ε_{12}^{v} , for example, has the form

$$\varepsilon^{v}_{12}(t) = \frac{1}{1+R}\{[G(\tau_1)]^{1/n}(t_1) + \ldots$$
$$+ [G(\tau_{m-1})]^{1/n}(t_{m-1}-t_{m-2}) \qquad (21)$$
$$+ [G(\tau_m)]^{1/n}(t-t_{m-1})\}^{n} \quad , \quad t_{m-1} < t \; .$$

where $F(\sigma_i)$ and $G(\tau_i)$ represent $F(\sigma_i-\sigma^*)$ and $G(\tau_i-\tau^*)$, see (4) and (5).

Total Strain:

The total strain following a series of steps or jumps in stress is found by adding to the elastic strain corresponding to the stresses existing at the time of interest the recoverable strain given by (13) or (14) and the nonrecoverable strain (19) or (21) for axial strain or shear strain, respectively.

The above approach [the viscous-viscoelastic theory (VV)] was employed to calculate the creep behavior corresponding to several complex stress histories and compared with actual results in the following section.

In addition, the strain hardening theory alone (SH) as described by (21) was employed also to predict the total creep strain. In this case the coefficient $\frac{1}{1+R}$ in (21) was replaced by unity and ε^{ve} was taken to be zero.

EXPERIMENTAL RESULTS AND COMPARISONS

Using the material constants in (2), (3), (4), and (5) determined from constant stress creep and recovery tests as described in [1] and given in Table I, creep resulting from step-up, step-down and recovery stress change experiments were predicted using the procedures described above. The results were then compared with corresponding experimental results as shown in Figures 1 through 4. The experiments consisted of tension or torsion creep tests in which abrupt changes in load were made at intervals. Several types of load changes often were made in the same experiment. In the following the predictions for similar types of load changes are compared with experiments rather than discussing the results of each testing sequence. The predictions based on (13), (14), (19) and (20), the viscous-viscoelastic (VV) theory, are shown as dot-dash lines. The short-dash lines represent the predictions based on strain hardening (SH) alone. The solid lines represent the predictions based on modifications of the viscous-viscoelastic (MVV) theory which are discussed in later paragraphs. In Fig. 1-4 omission of the dot-dash line or the dash line for any period indicates that the prediction based on the omitted theory is the same as that represented by the solid lines.

Step-up Experiments:

Step-up experiments are shown in Fig. 1 through 4. In Fig. 1 there is a sequence of two upward steps following the first period of creep. An upward step was preceded by a downward step to zero stress in Fig. 2 and 3. A small stress reduction preceded the step-up in Fig. 4.

(VV) Theory:

Except for a vertical displacement, the agreement between experiment and creep predicted by the (VV) theory is excellent for the second period in Fig. 1. During the third period the actual creep rate was somewhat greater than predicted by the (VV) theory and there was more of a "primary" type creep (greater rate of change of slope) than predicted.

The third period in Fig. 2 (involving reloading to a higher stress than the first loading) shows excellent agreement between the prediction of the (VV) theory and the test data taken from [2]. The third and fourth periods in Fig. 3A consist of reloading to the same stress as the first after a period at zero stress and then a step-up in stress. Again there is excellent agreement between data and prediction of the (VV) theory. The experiment in Fig. 4 involves creep at one stress followed by a small reduction in stress and then a reapplication of the same stress. In the third period the character of the creep curve and that predicted by the (VV) theory differ in that the primary-type behavior predicted at the start of the period was not observed. Also, the rate of creep was greater than predicted.

(SH) Theory:

For step-up experiments the predictions using the strain hardening (SH) theory are about the same as that of the viscous-viscoelastic (VV) theory. For the first period of loading, both theories yielded identical results. In Fig. 1, periods 2 and 3, and Fig. 4, period 3, the results from the strain hardening (SH) theory are somewhat closer to the test data than the (VV) theory. In Fig. 2, period 3, and Fig. 3A, period 4, the reverse is true. In Fig. 1 the primary type behavior of the (VV) theory at the start of the period is not found in the (SH) theory.

Recovery (Complete Unloading):

Recovery following unloading to zero stress is shown in Fig. 1 through 4. Agreement between the experimental data and the prediction of the (VV) theory is very good for all experiments except for small vertical shifts in Fig. 1 and 4. In all cases the shape is satisfactorily predicted.

Similar results were also found for recovery following three tests having complex histories of combined tension and torsion (to be reported later). The recovery data in the second period of Fig. 2 and 3 is not a prediction, however, as these data were used in [1] as input in obtaining the constants in Table I. In Fig. 1 and 4 the recovery shown in periods 6 and 4, respectively, followed a complicated history of changes in magnitude of stress.

Predictions of recovery from the strain hardening (SH) theory in all cases are incorrect. The strain hardening theory predicts no recovery upon complete unloading, although the experimental data in all cases show time-dependent recovery as predicted by the viscous-viscoelastic (VV) theory.

Step-down Stress Change (Partial Unloading):

Step-down experiments involving partial unloading are shown in Fig. 1 at periods 4 and 5, Fig. 3B at periods 5 through 8, and Fig. 4 at period 2. The changes in stress are about 17% in Fig. 1, 10% in Fig. 3, and 7% in Fig. 4. In all of these cases the prediction based on the (VV) theory showed a recovery-type of behavior, that is, a negative slope of the creep curve with a gradually reducing rate. In Fig 4. at period 2 the gradually reducing rate was reversed in the middle of the period. However, in every instance the observed creep behavior showed no negative rate, but a nearly constant small positive rate. On the other hand, as noted above, complete unloading to zero stress resulted in a recovery type curve (negative creep rate) in both the observed recovery and the prediction from the (VV) theory.

The prediction based on the (SH) theory for the step-down experiments showed a small positive rate which was quite similar to the form of the observed creep behavior. In Fig. 3 periods 5 through 8 the predictions for both (VV) and (SH) theories were about the same.

Discussion:

The following features of the above results were noted. (a) The strain hardening (SH) theory did not predict the recovery observed on complete removal of a stress component. (b) The creep rate following an increase in stress in all cases was somewhat greater than predicted. Since the contribution of the nonrecoverable component was about twice that of the recoverable component for the (VV) theory, it may be concluded that this is a defect of the work-hardening approach used in computing the nonrecoverable component of strain. (c) In the third period of Fig. 1, the data showed more of a "primary" type behavior than predicted. However, there is no such defect under similar circumstances in

period 4 of Fig. 3a. This may also be a defect of the work hardening concept.
(d) In the step-up tests and recovery at zero stress, there is no ambiguity as to how the creep limit enters into the calculation. However, on partial unloading the role of the creep limit is less clear.

MODIFICATION OF CONSTITUTIVE EQUATIONS,
(MVV) Theory

Some of the features of the step-down and recovery experiments not properly described by the (VV) or (SH) theories are better described by assuming that the behavior with regard to the creep limits is different for the nonrecoverable strain ε^{v} than for the recoverable strain ε^{ve} as follows:

(A) For the nonrecoverable strain component, the strain hardening rule is still applicable. Upon reduction of stress, this strain rate $\dot{\varepsilon}^{v}$ continues at the reduced but increasing rate prescribed by the strain hardening rule, (19), (20) and (21) for example, until the current stress σ_a equals or is less than the creep limit σ^*. When $\sigma_a \leq \sigma^*$, $\dot{\varepsilon}^{v}$ is zero as prescribed by (19), (20) and (21). Upon reloading to a stress above the creep limit, the nonrecoverable strain rate $\dot{\varepsilon}^{v}$ resumes at the rate prescribed by the same equations as though there had been no interval t_x for which $\sigma_a \leq \sigma^*$.

(B) For the recoverable strain components ε^{ve}, on partial unloading, the recoverable strain rate $\dot{\varepsilon}^{ve}$ becomes and remains zero for all reductions of stress until the total change in stress from the highest stress σ_{max} previously encountered to the current stress σ_a equals in magnitude the creep limit σ^*. That is,

$$\dot{\varepsilon}^{ve} = 0 \text{ when } (\sigma_{max} - \sigma_a) \leq \sigma^* \quad . \tag{22}$$

Equation (22) can be considered as meaning that for a small unloading, the recoverable strain component is "frozen" until the stress differential is greater than σ^* before the recovery mechanism is activated.

Besides the response that $\dot{\varepsilon}^{ve} = 0$ under the stress condition described by (22) for the (MVV) theory, there are two other possible responses for $\dot{\varepsilon}^{ve}$ under the stress condition given by (22): (a) $\dot{\varepsilon}^{ve} < 0$ (this has been covered by the (VV) theory) and (b) $\dot{\varepsilon}^{ve} > 0$ (for small partial unloading this is not admissible).

(C) For large partial unloading, $(\sigma_{max} - \sigma_a) > \sigma^*$, the recovery mechanism becomes active and the recoverable strain component ε^{ve} may be computed as if the previous stress continued to cause creep and a reverse stress equal to $(\sigma_{max} - \sigma_a)$ was applied to the specimen. The recoverable strain may be computed by the modified superposition principle except that the stress is replaced by the stress difference minus σ^* when the stress is reduced. This satisfies the requirement of complete recoverability of ε^{ve} upon complete unloading for one step loading only. Further load changes may involve difficulties because of nonlinearity.

(D) Upon increasing the stress to σ_b, $(\sigma_b \geq \sigma_a)$ following a period t_x (a dead zone) for which $(\sigma_{max} - \sigma_a) < \sigma^*$ and $\dot{\varepsilon}^{ve} = 0$ as discussed in (B) above, the recoverable strain component ε^{ve} continues in accordance with the viscoelastic behavior (12) as though the period t_x never occurred. Thus, in computing the behavior for situations described in (B) and (D), it is necessary to introduce a time shift in equations (13), (14) and (15) to eliminate the appropriate period t_x when ε^{ve} is "frozen." Thus the new time t' subsequent to a period $t_x = (t_b - t_a)$, becomes $t' = t - (t_b - t_a)$, where t is the real time and t_a, t_b are the times when σ_a, and σ_b are applied.

(E) Of course it is possible if not probable that the creep surface in stress space defining the creep limit changes size, shape and position as a result of plastic and creep strains. However, the nature of such changes, if any, is not known at present.

The predictions of the modified viscous-viscoelastic (MVV) theory computed in accordance with A through D above are shown as solid lines in Fig. 1 through 4. These predictions are in accord with the experimental data in Fig. 1, 2, 3A and 4, and are generally better representations of the material behavior than either the (VV) or (SH) theories.

However, the small step-down experiments shown in Fig. 3B are best represented by the (VV) theory with the (SH) theory yielding the next best description of the data. The data in Fig. 3B are an approximation of stress relaxation in that the stress was held constant at each step until the strain had returned to its previous value before the stress was reduced again. Also the shape of the actual recovery curve resulting from complete unloading following a series of unloading steps (Fig. 1 period 6) is better described by the (VV) than the (MVV) theory. A similar result was also observed in the recovery following complete unloading in a creep test under variable combined tension and torsion (to be reported later).

All the partial step-down tests shown in Fig. 1 and 4 are in the range where the change in stress is less than the magnitude of the creep limit (hence there was no contribution from ε^{ve}) and the stress following the change was greater than the creep limit (hence $\dot{\varepsilon}^{v}$ would continue at a reduced rate). As shown for partial step-down tests there was no "recovery" type behavior and the creep rate was positive or approaching zero, which was in accord with the (MVV) theory. Additional step-down tests in which the change in stress is greater than the magnitude of the creep limit are needed to explore further the role of the creep limit.

AGING

The possibility that aging may have affected the results of the experiments was investigated further. A tension creep test was performed at 25 ksi stress at 200°C (392°F) after aging at the same temperature for 95 hr. The results showed a small increase in creep rate compared to the results of Tests F1 and 16 reported in [1] at the same stress but aged for 18 hr. Analysis of the data yielded the following values of the constants in (1) for the test which was aged for 95 hr.: For best fit $n = 0.237$, $\varepsilon^o = 0.2668$ per cent, $\varepsilon^+ = 0.0557$ per cent hr^{-n}; for $n = 0.270$, $\varepsilon^o = 0.2716$, $\varepsilon^+ = 0.0508$. The creep rate $\dot{\varepsilon}$ at 1 hr. is given by $n\varepsilon^+$. Making this computation for the three ages available yielded the following creep rates: Aged 18 hr., the average of Tests F1 and 16 yielded $\dot{\varepsilon} = 0.0085$%/hr.; aged 95 hr. $\dot{\varepsilon} = 0.0132$%/hr., aged 1103 hr., $\dot{\varepsilon} = 0.230$%/hr. Interpolating these values on the basis of either a log-log relation or linear time-log strain-rate relation yielded an increase of creep rate from 18 to 30 hr. of about 7 per cent for either interpolation. Thus during the testing time of the experiments reported the creep rate increased about 1/2 per cent per hr., which is considered negligible over the time span of the experiments.

RESULTS AND CONCLUSIONS

Analysis of results of creep tests of 2618 aluminum under a variety of changes in stress during creep in the nonlinear range show that a strain hardening (SH) theory does not properly describe the behavior on unloading or reloading; but a viscous-viscoelastic theory with certain modifications (MVV) theory predicts most of the features of the observed creep behavior quite well.

Among the conclusions are the following:

1. The behavior may be represented by resolving the time-dependent strain into recoverable and nonrecoverable components having the same time dependence.

2. The material behaves as though there was a creep limit such that creep is very small or zero unless the stress is greater than a limiting value.
3. On partial unloading the material behaves as though the nonrecoverable strain component ε^{v} continued to creep in accordance with strain hardening unless the stress became less than the creep limit; whereas the recoverable strain component ε^{ve} remained constant unless the decrease in stress exceeded the magnitude of the creep limit.
4. On reloading following an interval t_x of partial unloading involving no further change in ε^{ve} the component ε^{ve} resumed creep as though the interval t_x did not exist.
5. Very small reductions of stress are best represented by the viscous-visco-elastic (VV) theory, which is inconsistent with the behavior under small stress reductions.
6. Recovery on complete unloading following a history of step changes in stress is reasonably represented by the (VV) or (MVV) theories, but best represented by the (VV) theory.

ACKNOWLEDGMENT

This work was supported by the Office of Naval Research and the Army Research Office, Research Grant No. DAAG29-78-G-0185. The material was contributed by the Aluminum Company of America. The authors are grateful to: U. W. Cho for helpful discussion and recomputation of theory; R. M. Reed for performing some of the experiments and M. C. Gingrich for typing the manuscript.

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Table I. Constants for Equations (2), (3), (4), and (5).

F_i^+ , G_i^+
$F_1^+ = 6.084 \times 10^{-12}$, per $Pa\text{-}hr^n$ (0.004195, % per $ksi\text{-}hr^n$)
$F_2^+ = -7.431 \times 10^{-20}$, per $Pa^2\text{-}hr^n$ (-0.0003533, % per $ksi^2\text{-}hr^n$)
$F_3^+ = 7.596 \times 10^{-28}$, per $Pa^3\text{-}hr^n$ (0.0000249, % per $ksi^3\text{-}hr^n$)
$\sigma^* = 9.143 \times 10^7$, Pa (13.26, ksi)
$G_1^+ = 7.170 \times 10^{-12}$, per $Pa\text{-}hr^n$ (0.004944, % per $ksi\text{-}hr^n$)
$G_2^+ = 2.703 \times 10^{-28}$, per $Pa^3\text{-}hr^n$ (0.00000886, % per $ksi^3\text{-}hr^n$)
$\tau^* = 4.571 \times 10^7$, Pa (6.630, ksi)

Note: $n = 0.270$, $R = 0.55$.

FIGURE CAPTIONS

Fig. 1. Creep of 2618AL at 200°C Under Step Loading. Where a theory is not shown it is the same as the (MVV) Theory. Numbers indicate periods.

$$\tau_1 = 69.0 \text{ MPa } (10 \text{ ksi}) ,$$
$$\tau_2 = 82.7 \text{ MPa } (12 \text{ ksi}) ,$$
$$\tau_3 = 96.5 \text{ MPa } (14 \text{ ksi}) .$$

Fig. 2. Creep of 2618AL at 200°C Under Complete Unloading and Reloading to a Higher Stress. Where a theory is not shown it is the same as the (MVV) Theory. Numbers indicate periods.

$$\sigma_1 = 137.9 \text{ MPa } (20 \text{ ksi}) ,$$
$$\sigma_2 = 193.1 \text{ MPa } (28 \text{ ksi}) .$$

Fig. 3A. Creep of 2618AL at 200°C Under Complete Unloading, Reloading and Step-Up. Where a theory is not shown it is the same as the (MVV) Theory. Numbers indicate periods.

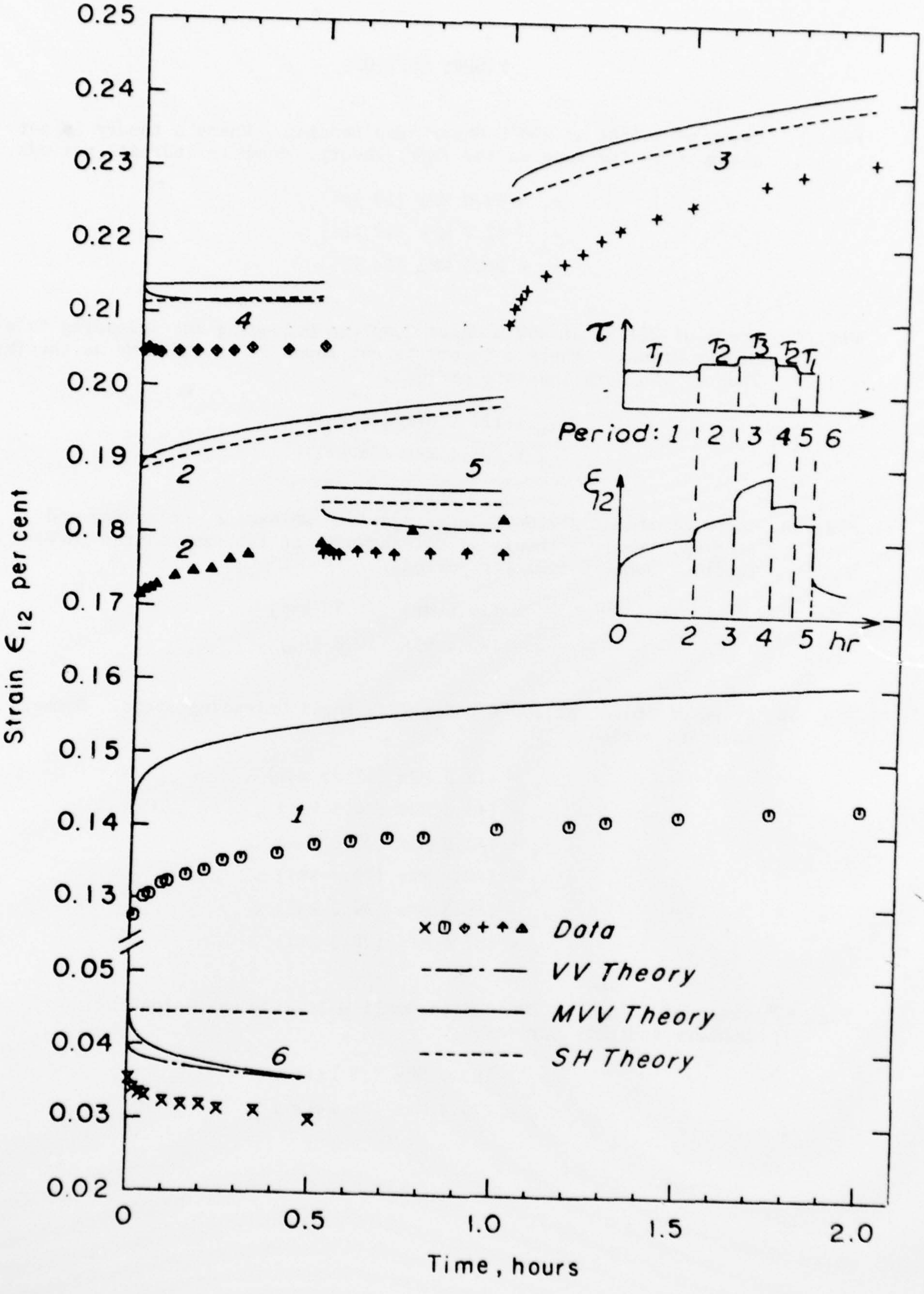
$$\sigma_1 = 119.5 \text{ MPa } (17.33 \text{ ksi}) ,$$
$$\sigma_2 = 143.4 \text{ MPa } (20.8 \text{ ksi}) .$$

Fig. 3B. Creep of 2618AL at 200°C Under Very Small Unloading Steps. Numbers indicate periods.

$$\sigma_1 = 119.5 \text{ MPa } (17.33 \text{ ksi}) ,$$
$$\sigma_2 = 143.4 \text{ MPa } (20.8 \text{ ksi}) ,$$
$$\sigma_3 = 142.0 \text{ MPa } (20.6 \text{ ksi}) ,$$
$$\sigma_4 = 140.7 \text{ MPa } (20.4 \text{ ksi}) ,$$
$$\sigma_5 = 139.3 \text{ MPa } (20.2 \text{ ksi}) ,$$
$$\sigma_6 = 137.9 \text{ MPa } (20.0 \text{ ksi}) .$$

Fig. 4. Creep of 2618AL at 200°C With Small Unloading and Reloading. Numbers indicate periods.

$$\sigma_1 = 193.1 \text{ MPa } (28 \text{ ksi}) ,$$
$$\sigma_2 = 179.4 \text{ MPa } (26 \text{ ksi}) .$$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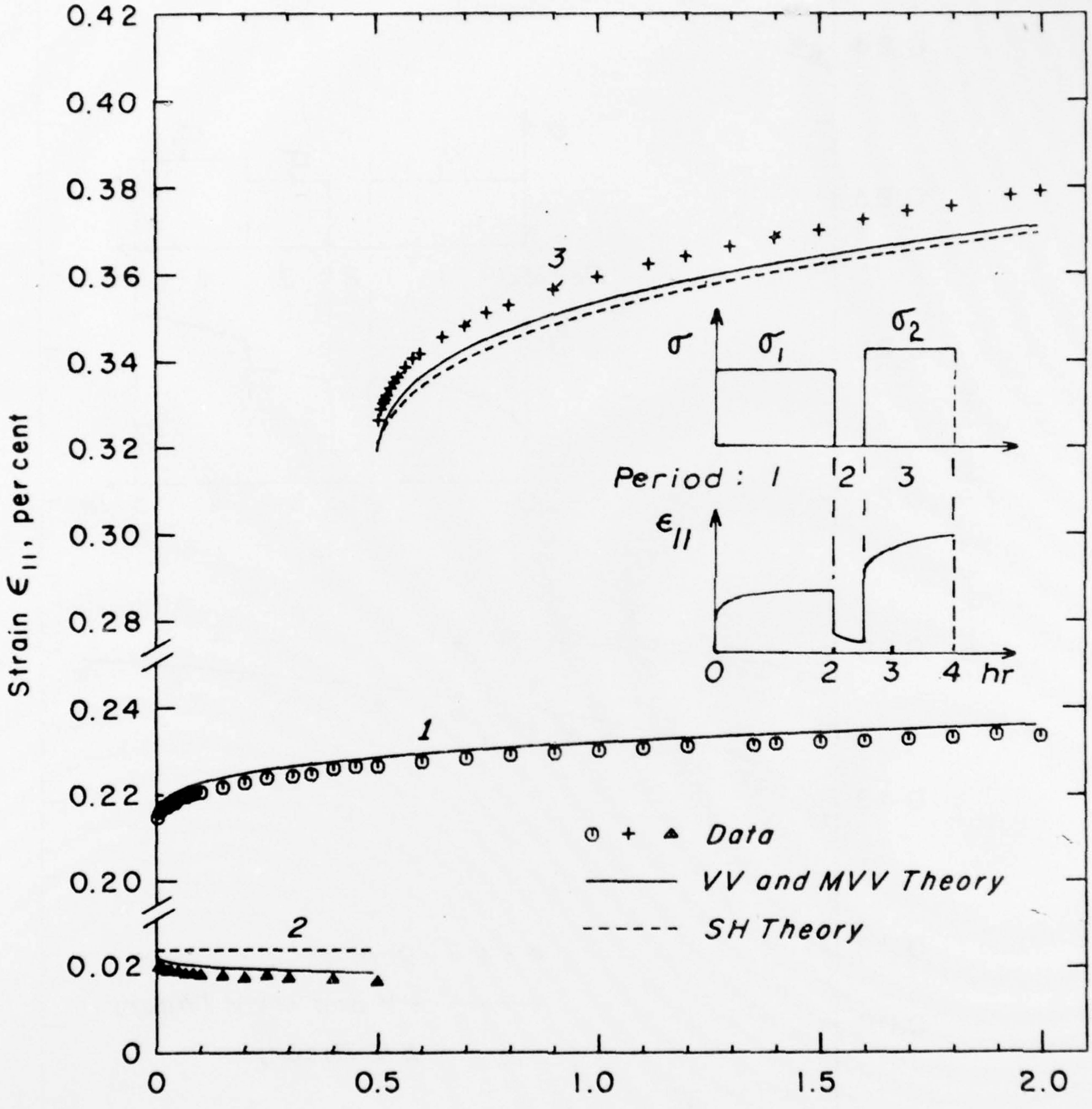
0.24
0.23
0.22
0.21
0.20
0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.05
0.04
0.03
0.02
Strain ϵ_{12} per cent
0
0.5
1.0
1.5
2.0
Time, hours
1
2
3
4
5
6
Data
VV Theory
MVV Theory
SH Theory
τ
τ_1 τ_2 τ_3 τ_2 τ_1
Period: 1 2 3 4 5 6
ϵ_{12}
0 2 3 4 5 hr

Fig. 1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0.40
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0.30
0.28
0.24
0.22
0.20
0.02
0
Strain ϵ_{11}, per cent
0
0.5
1.0
1.5
2.0
Time, h
1
2
3
σ
σ_1
σ_2
Period : 1 2 3
ϵ_{11}
0 2 3 4 hr
Data
VV and MVV Theory
SH Theory

Fig. 2

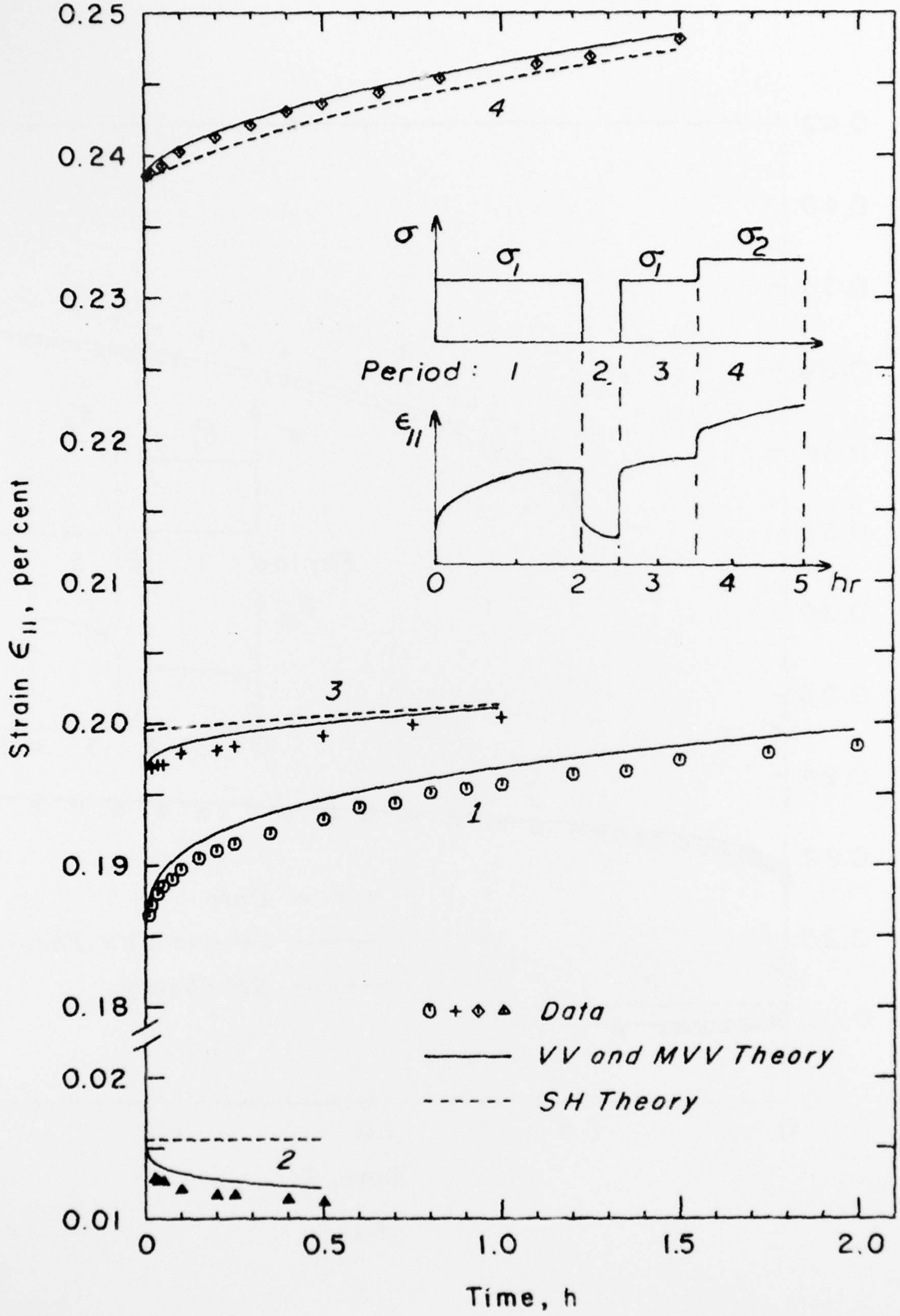
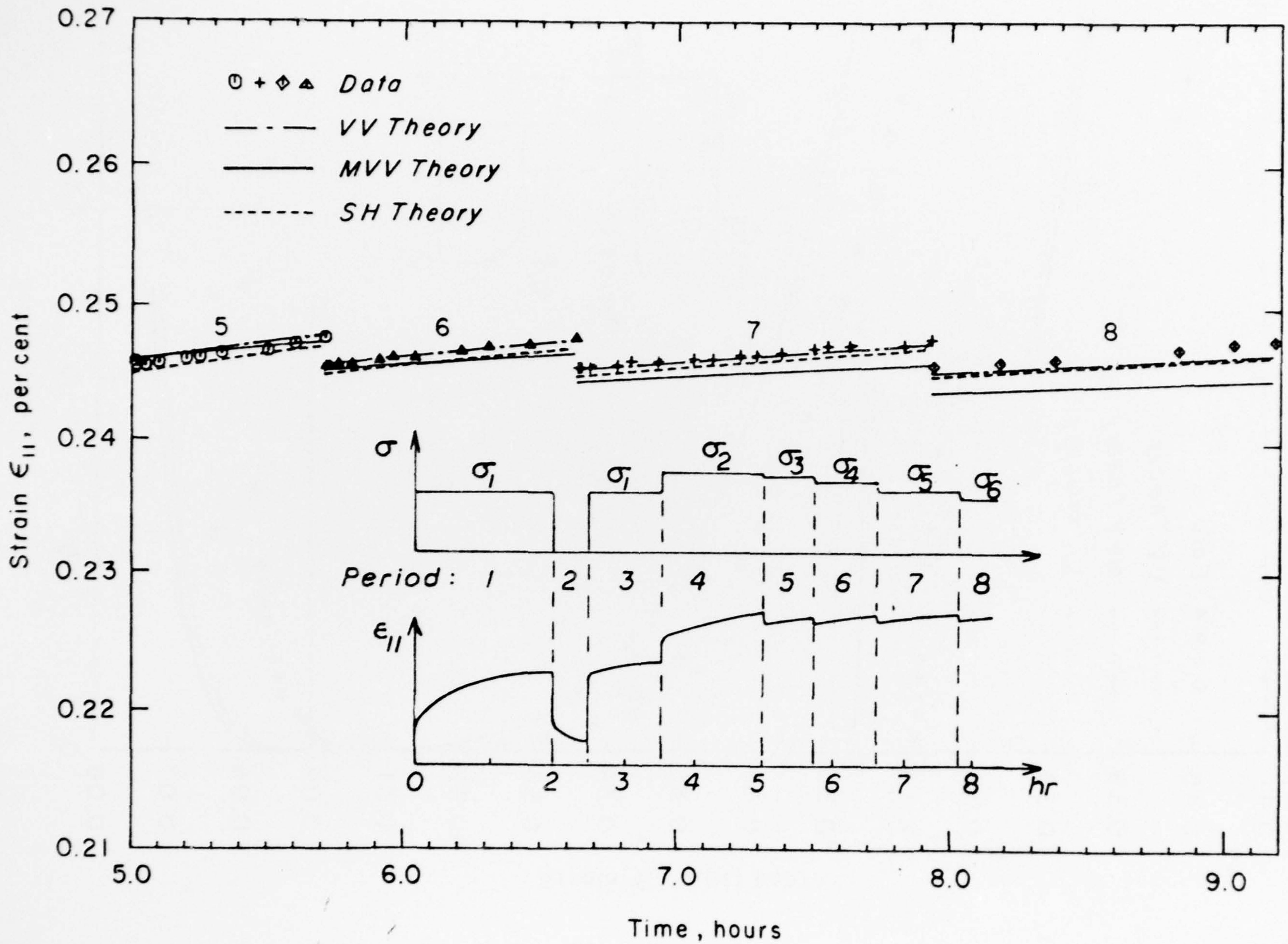

0.25
0.24
0.23
0.22
0.21
0.20
0.19
0.18
0.02
0.01
Strain ϵ_{11}, per cent
0
0.5
1.0
1.5
2.0
Time, h
4
3
1
2
σ
σ_1
σ_1
σ_2
Period: 1 2 3 4
ϵ_{11}
0 2 3 4 5 hr
Data
VV and MVV Theory
SH Theory

Fig. 3A



0.27
0.26
0.25
0.24
0.23
0.22
0.21
Strain ϵ_{11}, per cent
5.0
6.0
7.0
8.0
9.0
Time, hours
Data
VV Theory
MVV Theory
SH Theory
5
6
7
8
σ
σ_1
σ_1
σ_2
σ_3
σ_4
σ_5
σ_6
Period: 1 2 3 4 5 6 7 8
ϵ_{11}
0 2 3 4 5 6 7 8 hr

Fig. 3B

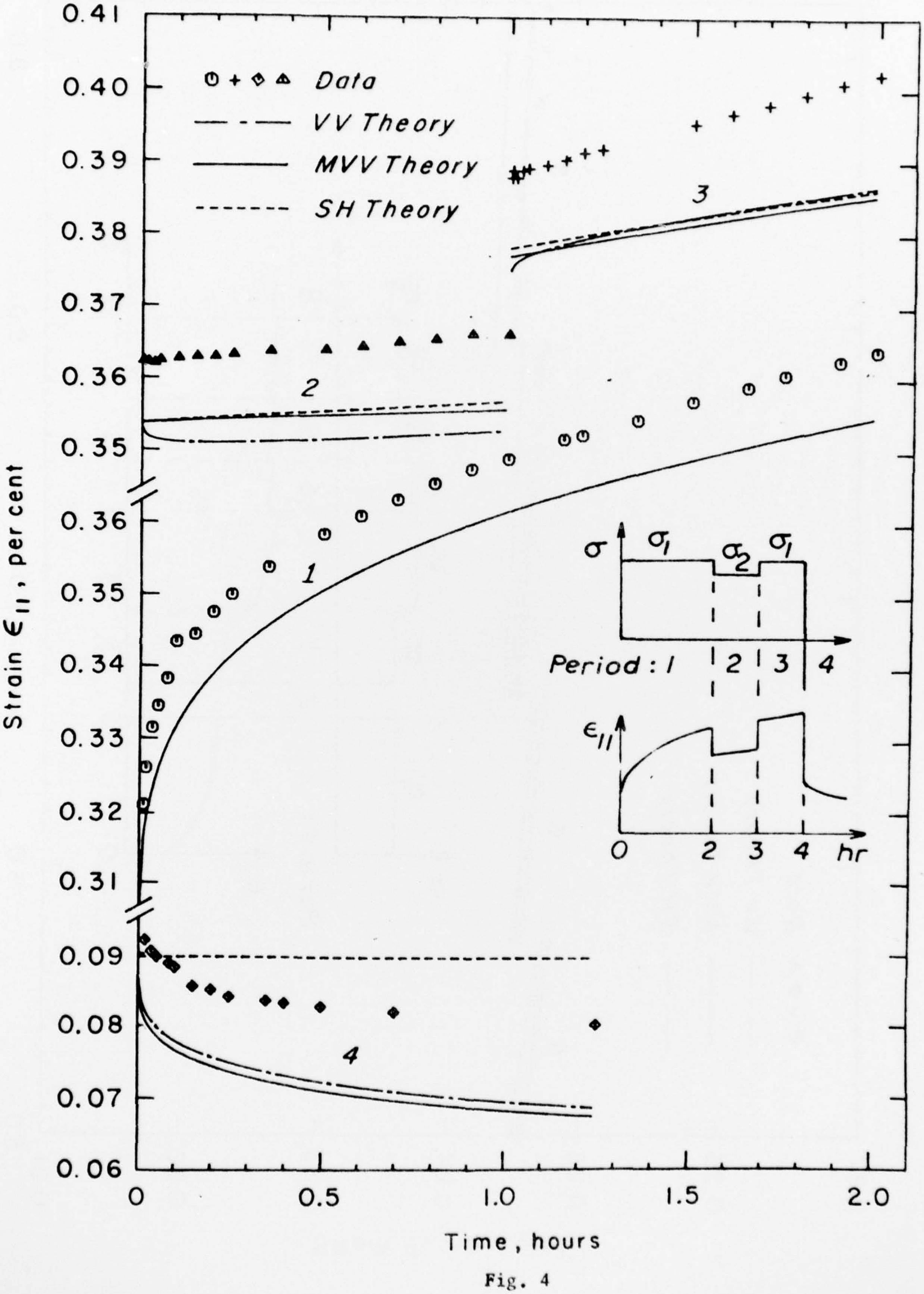

Data
VV Theory
MVV Theory
SH Theory
Strain ϵ_{11}, per cent
Time, hours
Period: 1 2 3 4
hr

Fig. 4